MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 3, 2022/2023

DMA5101 – MATHEMATICAL TECHNIQUES 1

(TC1L)

2 OCTOBER 2023
9.00 a.m. – 11.00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENT

1. This question paper consists of **6** pages (**5** pages questions and **1** page of Appendix). Key formulae are given in the Appendix.

2. Answer ALL questions.

3. Please shade your answers for Part A in the OMR sheet provided.

4. Write all your answers for Part B in the Answer Booklet provided. All necessary working steps must be shown.

PART A: 25 MULTIPLE CHOICE QUESTIONS **[25 MARKS]**

Instruction: Shade your answers in the OMR sheet provided.

Use this information to answer Question 1 and 2.
Given $f(x) = x(x+3)^2$.

1) By using the leading Coefficient Test, determine the end behavior of the graph of $f(x)$.
   A) The graph falls to the left and rises to the right.
   B) The graph rises to the left and falls to the right.
   C) The graph falls to the left and right.
   D) The graph rises to the left and right.

2) From the function, determine the correct multiplicity of each zeros of the function.
   A) At $x = 0$, the graph crosses the x-axis.
      At $x = -3$, the graph crosses the x-axis.
   B) At $x = 0$, the graph crosses the x-axis.
      At $x = -3$, the graph touches the x-axis and turn around.
   C) At $x = -3$, the graph crosses the x-axis.
      At $x = 0$, the graph touches the x-axis and turn around.
   D) At $x = -3$, the graph touches the x-axis and turn around.
      At $x = 0$, the graph touches the x-axis and turn around.

Use this information to answer Question 3 – 5.
Given $f(x) = \frac{x}{x^2+2}$

3) Determine the domain for $f(x)$.
   A) $(-\infty, \infty)$
   B) $(-\infty, \sqrt{2}) \cup (\sqrt{2}, \infty)$
   C) $(-\infty, -2) \cup (-2, \infty)$
   D) $(-\infty, -\sqrt{2}) \cup (-\sqrt{2}, \sqrt{2}) \cup (\sqrt{2}, \infty)$

4) Determine the vertical asymptote(s) for $f(x)$.
   A) $x = 2$
   B) $x = 0$
   C) $x = 2, x = 0$
   D) No vertical asymptote

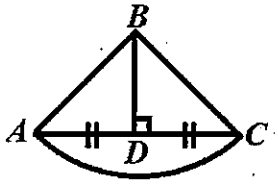
5) Determine the horizontal asymptote for $f(x)$.
   A) $y = 1$
   B) $y = 0$
   C) $y = 2$
   D) No horizontal asymptote

6) Find $\tan\theta$, given that $\cos\theta = \frac{2}{5}$ and $\csc\theta > 0$.
   A) $\frac{5}{\sqrt{21}}$
   B) $\frac{\sqrt{21}}{5}$
   C) $\frac{2}{\sqrt{21}}$
   D) $\frac{\sqrt{21}}{2}$

Continued...

Use the information below to answer Question 7 and 8.

The diagram below shows sector ABC with center B, two right angle triangles of ABD and CBD where the length of line $AD = DC$ and a segment ADC. Given that the radius and the length of arc of the sector is 12cm and 32cm, respectively.



7) Find the angle ABC, in degree.

A) 21.49° B) 27.21° C) 152.79° D) 207.21°

8) If the angle BCD and BAD have the same value, calculate the line segment of ADC.

A) 11.66 B) 12.34 C) 23.33 D) 24.69

Use the information below to answer Question 9 and 10.

Given $w = -2 + i2\sqrt{3}$.

9) Convert w into polar form, for $r > 0,\ 0° \leq \theta \leq 360°$.

A) $4(\cos(-60°) + i\sin(-60°))$ B) $4(\cos 120° + i\sin 120°)$

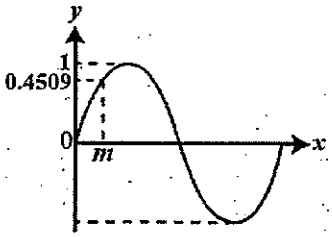
C) $4(\cos 60° + i\sin 60°)$ D) $4(\cos 240° + i\sin 240°)$

10) Given $z = 2(\cos(30°) + i\sin(30°))$, compute $\frac{w}{z}$.

A) $2(\cos(-90°) + i\sin(-90°))$ B) $2(\cos 90° + i\sin 90°)$

C) $2(\cos 30° + i\sin 30°)$ D) $2(\cos 4° + i\sin 4°)$

11) The diagram below shows the graph of $y = \sin(x°)$. Find the value of $\cos(m)$.



A) 0.4678 B) 0.7834

C) 0.8926 D) 0.9001

Use the information below to answer Question 12 – 14.

The trigonometric function, $y = 2\cos(2x - 2\pi)$ has been through several transformation from $f(x) = \cos(x)$.

12) Identify the phase shift of the function, y.

A) $-\pi$ B) π C) 2π D) 3π

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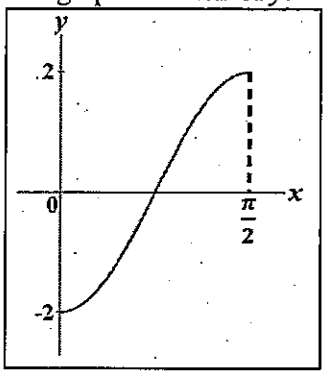
13) Identify the period of the function, y.

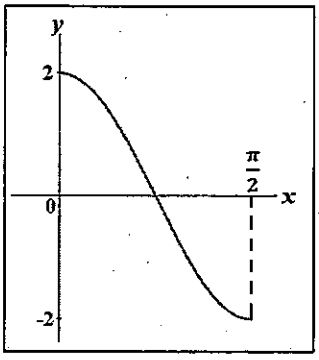
A) $-\pi$ B) π C) 2π D) 3π

14) Identify the graph function of y.

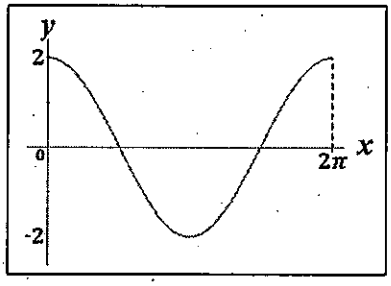
A)



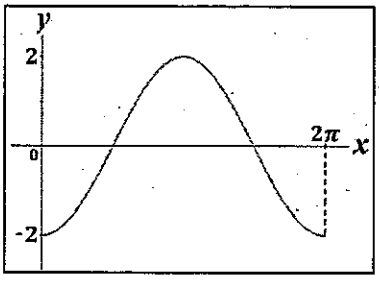
B)



C)



D)



15) Identify the appropriate interval for one complete period for $y = 4\tan\left(x - \frac{\pi}{4}\right)$.

A) $-\frac{\pi}{4} < x < \frac{\pi}{4}$ B) $-\frac{\pi}{2} < x < \frac{3\pi}{4}$

C) $-\frac{\pi}{4} < x < \frac{3\pi}{4}$ D) $-\frac{3\pi}{4} < x < \frac{3\pi}{4}$

Use the information below to answer Question 16 and 17.

Given the matrix $A = \begin{bmatrix} -3 & -8 \\ 7 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 6 \\ -9 & -4 \end{bmatrix}$.

16) Compute $3A + 2B$

A) $\begin{bmatrix} -4 & -2 \\ -2 & 1 \end{bmatrix}$ B) $\begin{bmatrix} -11 & 3 \\ -12 & 7 \end{bmatrix}$

C) $\begin{bmatrix} -4 & -12 \\ -3 & 7 \end{bmatrix}$ D) $\begin{bmatrix} -11 & -12 \\ 3 & 7 \end{bmatrix}$

17) Find matrix X where $B - X = A^T$.

A) $\begin{bmatrix} 2 & -1 \\ -1 & -9 \end{bmatrix}$ B) $\begin{bmatrix} -4 & -2 \\ -2 & 1 \end{bmatrix}$

C) $\begin{bmatrix} -4 & 13 \\ -17 & 1 \end{bmatrix}$ D) $\begin{bmatrix} 2 & -16 \\ 15 & -9 \end{bmatrix}$

Continued...

Use the information below to answer Question 18 and 19.

The inverse matrix of $\begin{bmatrix} 2p+2 & 15 \\ 15 & 4q \end{bmatrix}$ is $\begin{bmatrix} -16 & 15 \\ 15 & -14 \end{bmatrix}$ where p and q are constant.

18) Determine the value of p.

A) -9 B) -6 C) 6 D) 9

19) Determine the value of q.

A) -4 B) -3 C) 3 D) 4

20) Compute the matrix product of $\begin{bmatrix} -9 & 8 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 6 & -3 \end{bmatrix}$.

A) $\begin{bmatrix} 12 & -21 \\ 58 & 50 \end{bmatrix}$ B) $\begin{bmatrix} 12 & -69 \\ 58 & 20 \end{bmatrix}$

C) $\begin{bmatrix} 84 & 69 \\ 58 & 50 \end{bmatrix}$ D) $\begin{bmatrix} 84 & 21 \\ 58 & 20 \end{bmatrix}$

Use the information below to answer Question 21 – 23.

Given vectors $\boldsymbol{a} = \langle -5,7,8 \rangle$ and $\boldsymbol{b} = \langle 2,-3,1 \rangle$.

21) Compute $2\boldsymbol{a} + 3\boldsymbol{b}$.

A) $\langle -4,5,19 \rangle$ B) $\langle -2,4,9 \rangle$ C) $\langle -4,8,19 \rangle$ D) $\langle -5,5,19 \rangle$

22) Compute $|2\boldsymbol{a} + 3\boldsymbol{b}|$.

A) 20.05 B) 10.05 C) 21.00 D) 20.27

23) Find the cross product of $\boldsymbol{a}$ and $\boldsymbol{b}$.

A) $31i - 21j + k$ B) $17i + 11j + 29k$

C) $31i + 21j + k$ D) $17i - 11j + 29k$

24) If vector $\boldsymbol{a} = 4i + 3j - mk$ and vector $\boldsymbol{b} = 9i - j + 7k$, find the value of m if a dot product of vector $\boldsymbol{a} \cdot \boldsymbol{b} = 12$.

A) -3 B) 3 C) 2 D) -2

25) The equation of a plane passing through a point $(3,1,7)$ and perpendicular to the vector $n = 2i + 4j - 5k$ is $2x + ay - bz + 25 = 0$ where a and b are constant. Find the values of a and b.

A) $a = 4, b = -5$ B) $a = -5, b = 4$

C) $a = 5, b = 4$ D) $a = 4, b = 5$

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PART B **[25 Marks]**

Question 1

a) Given $g(x) = 2x^2 - 4x - 6$.
   i. Determine whether the graph opens upward or downward. (1 mark)
   ii. Calculate the vertex point of the parabola. (2.5 marks)
   iii. Find y-intercept. (1.5 marks)

b) By using Pythagorean Identities, show that $\frac{\sin^2 x}{1-\cos x} = 1 + \cos x$. (1.5 marks)

c) Given that $z = 3(\cos 50° + i \sin 50°)$ is a complex number in polar form. Use De Moivre's Theorem to compute z^3 and simplify your answer in the standard form of $a + bi$. (2.5 marks)

d) Given the sinusoidal function of $y = 3\sin(2x - 4\pi) + 2$. Determine the amplitude, period, phase shift and vertical shift. (3 marks)

Question 2

a) Three teachers bought some colored pens at a bookstore. The table below shows the number of pens bought by the teachers. Let x, y and z be the price of the blue, black and red pen respectively. (8 marks)

Teachers	Item			Total price (RM)
	Blue Pen	Black Pen	Red Pen	
Julie	2	3	2	150
Beth	2	2	4	200
Sandra	5	4	2	250

   i. Represent the above information in the form of $AX = B$.
   ii. From part (i), calculate the price difference of blue and black pen.

b) Given matrix $A = \begin{pmatrix} 7 & -2 & 1 \\ -1 & 9 & 5 \\ 3 & -6 & 8 \end{pmatrix}$ where matrix $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$, find the cofactor of entry a_{33} (1.5 marks)

c) Given the two planes $4x + 3y + 2z = 1$ and $4y - 9z + 7x = 1$ are intersecting with one another, find the angle between them. (3.5 marks)

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APPENDIX

Pythagorean Identity:
$\sin^2\theta + \cos^2\theta = 1$

Trigonometric functions:
$y = A\sin(Bx - C)$ or $y = A\cos(Bx - C)$
amplitude $= |A|$, period $= \frac{2\pi}{B}$, and phase shift $= \frac{C}{B}$

Products and quotients in polar form:
$$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$$
$$\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$$

De Moivre's theorem:
$$z^n = r^n[\cos(n\theta) + i\sin(n\theta)]$$

Length of vector:
The length of the vector $\boldsymbol{a} = \langle a_1, a_2, a_3\rangle$ is $|\boldsymbol{a}| = \sqrt{a_1{}^2 + a_2{}^2 + a_3{}^2}$.

Dot Product
If θ is the angle between the vector $\boldsymbol{a} = \langle a_1, a_2, a_3\rangle$ and $\boldsymbol{b} = \langle b_1, b_2, b_3\rangle$, then
$\boldsymbol{a}\cdot\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3 = |\boldsymbol{a}||\boldsymbol{b}|\cos\theta$

Cross Product
If θ is the angle between the vector $\boldsymbol{a} = \langle a_1, a_2, a_3\rangle$ and $\boldsymbol{b} = \langle b_1, b_2, b_3\rangle$, then
$\boldsymbol{a}\times\boldsymbol{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1\rangle$

Area of triangle PQR $= \frac{1}{2}\left|\vec{PQ}\times\vec{PR}\right|$

Equation of lines
Vector equation: $\boldsymbol{r} = \boldsymbol{r}_0 + t\boldsymbol{v}$

Equation of planes
Scalar equations: $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$

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